# FALL SCHOOL

# Interactions between compressed sensing, random matrices and high dimensional geometry

## November 16-20, 2009

## University Paris-Est Marne-la-Vallée

## Assistant

Christiane Lafargue
Phone: +33(0)1.60.95.75.20
Fax: +33(0)1.60.95.75.45
christiane.lafargue@univ-mlv.fr
http://perso-math.univ-mlv.fr/users/banach/Fallschool2009/

# PROGRAM

## Djalil Chafai

*Behavior of the largest and smallest singular values of random matrices*

The extremal singular values of a matrix are very natural geometrical quantities concentrating an essential information on the invertibility and stability of the matrix. This short course of four hours aims to provide an accessible introduction to the notion of singular values of matrices and their behavior when the entries are random, including quite recent striking results from Random Matrix Theory. Gaussian models will serve as a backbone towards universality. Some electronic lecture notes will be hopefully available.

## Olivier Guédon

*Empirical methods and selection of characters*

Several problems of harmonic analysis are deeply related with the study ofsome empirical processes. Classical example of characters in $L_2$ are the Fourier or the Walsh system. In the 80's, deep results about the selection of characters that satisfy good analytic properties have been proved. We will present how these proofs are now understood, how they are connected with empirical processes and what are the connections with reconstruction of sparse signals.

## Guillaume Lecué

*Basic tools from empirical processes theory applied to the compress sensing problem*

The RIP condition is at the heart of the problem of reconstruction of spare vectors by the basis pursuit algorithm. Up to now, no deterministic matrix has been proved to satisfy this condition. Checking if some Ensemble of random matrices satisfy this condition can be seen as a problem of empirical processes theory. We will introduce some tools used in this theory to solve this kind of problem.

## Shahar Mendelson

### *Applications of chaining methods*

In the lectures I intend to present various structural results on typical coordinate projections of classes of functions. Structural results of this flavour have played a central role in empirical processes theory throughout the years and, as I will explain, are important in many statistical and geometric applications.

The main theme will be to show how chaining methods allow one to obtain rather accurate information on the random geometry of a given class of functions, and explain the role various metric structures endowed on the class have in the analysis.
One application I will present is a method of controlling the supremum of an empirical process indexed by powers of functions that are very poorly bounded (for example, powers of linear functional on R^n). I will explain why this is an essential component in many problems in Asymptotic Geometric Analysis and Nonparametric Statistics and how the structural methods come into the picture.

## Alain Pajor

### *The Restricted Isometry Property (RIP) of some models of random matrices and High dimensional Geometry*

Compressed Sensing is a quite new framework that enables approximate and exact reconstruction of sparse signals from incomplete measurements. The ideas and principles are strongly related to other problems of different fields such as approximation theory and Asymptotic Geometric Analysis.

It is not in our intention to give a course on compressed sensing, but preferably to put some emphasis on interactions in particular with asymptotic geometric analysis and random matrices. The possibility to reconstruct a class of vectors is highly related to its complexity and many tools were developed in Asymptotic Geometric Analysis to analyse different concepts of complexity of subsets of Banach spaces.

The Fall School will take place on the **campus Cité Descartes**. The best way to come to the campus is by RER. Take **line A of RER** (the red one) **direction Chessy/ Marne-la-vallée** or **Torcy**. Get off the train at the **station Noisy-Champs** (the best is to get off from the station at the head of the train when coming from Paris). The buildings (ESIEE, ENPC and Copernic) are 10 minutes walk from the train station Noisy-Champs (cf. the map « Campus de la cité Descartes »).

All the courses will take place either in:

1) **ESIEE** (l'école d'ingénieurs des sciences et technologies de l' information et de la communication) ;
2) **ENPC** (école nationale des ponts et chaussées);
3) The Buffet (Thursday lunch) will happen in the **Mathematics departement of Univeristé Marne-la-vallée** (located in **COPERNIC** building, level 4).



Plan d'accès à l'Université de Marne-la-Vallée
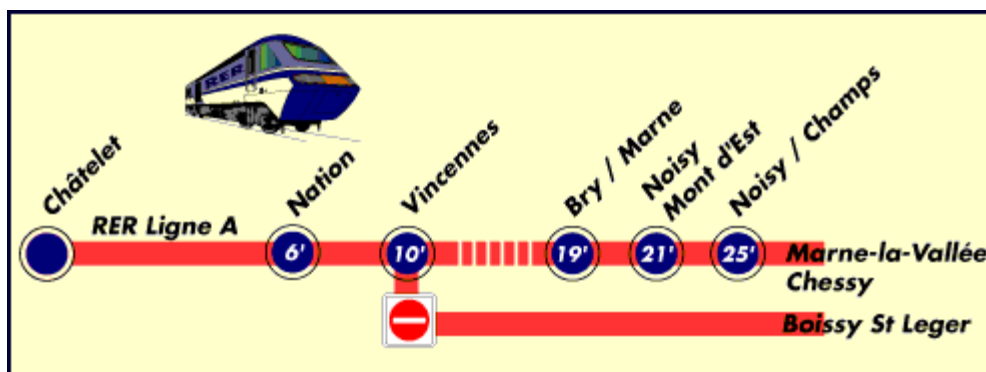
| Monday | ESIEE building (amphi 110 main floor) |
|---|---|
| 09:30-10:15 | Coffee |
| 10:15-10:30 | Opening remarks |
| 10:30-11:30 | Alain Pajor<br>*Introduction to Compressed sensing* |
| 11:30-11:45 | Coffee break |
| 11:45-12:45 | Djalil Chafai<br>*Behavior of the largest and smallest singular values of random matrices I* |
| 12:45-14:00 | Lunch at ESIEE |
| 14:00-15:30 | Guillaume Lecué<br>*Basic tools from empirical processes theory applied to the compress sensing problem I* |
| 15:30-16:00 | Tea break |
| 16:00-17:00 | Alain Pajor<br>*The RIP property for some models of random matrices I* |

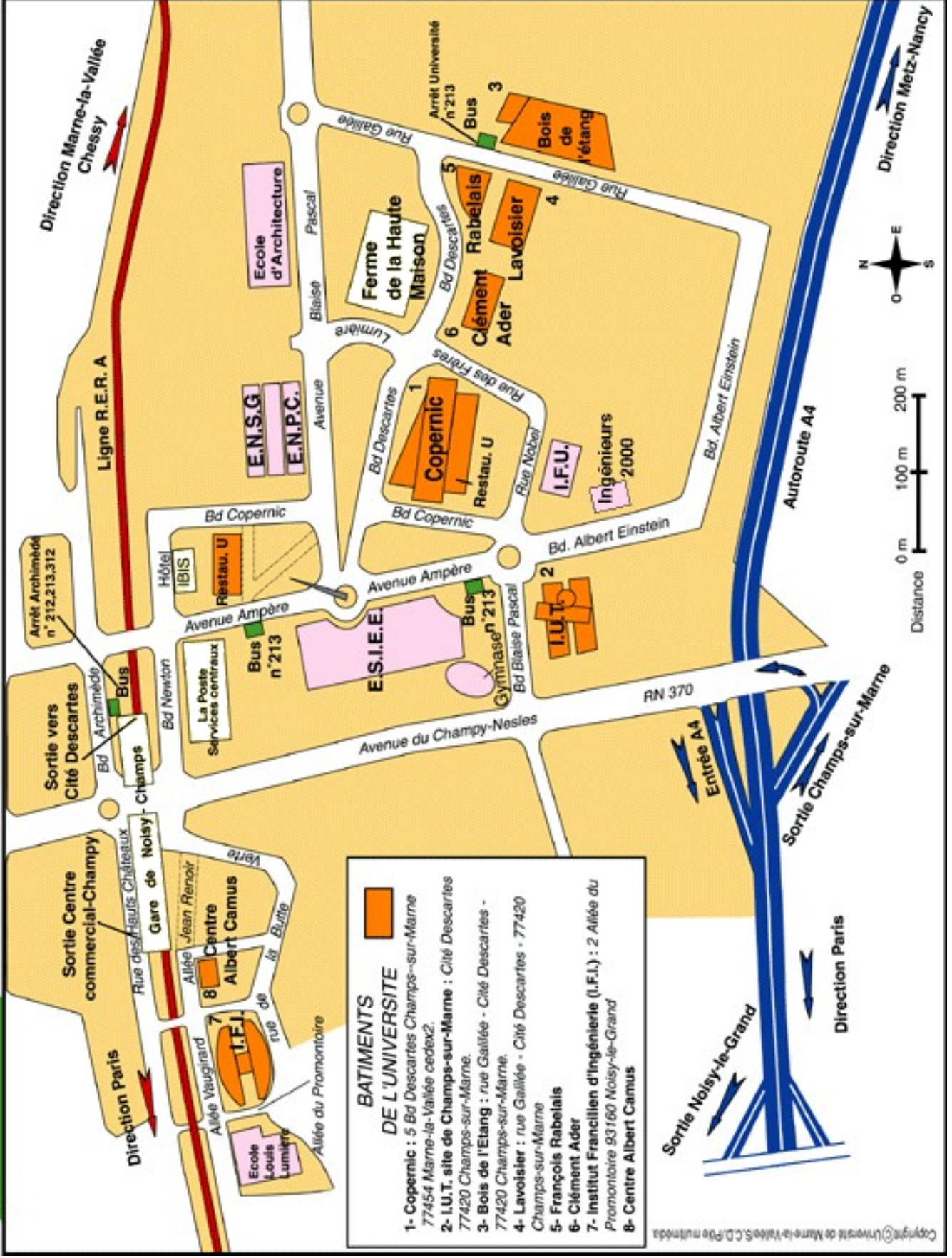| Tuesday | ESIEE building (amphi 110 main floor) |
|---|---|
| 09:30-10:00 | Coffee |
| 10:00-11:00 | Guillaume Lecué<br>*Basic tools from empirical processes theory applied to the compress sensing problem II* |
| 11:00-11:15 | Coffee break |
| 11:15-12:45 | Djalil Chafai<br>*Behavior of the largest and smallest singular values of random matrices II* |
| 12:45-14:00 | Lunch at ESIEE |
| 14:00-15:30 | Olivier Guédon<br>*Empirical methods and selection of characters I* |
| 15:30-16:00 | Tea break |
| 16:00-17:00 | Shahar Mendelson<br>*Applications of chaining methods* I |

| Wednesday | ESIEE building (amphi 110 main floor) |
|---|---|
| 09:30-10:00 | Coffee |
| 10:00-11:30 | *Shahar Mendelson*<br>*Applications of chaining methods II* |
| 11:30-11:45 | Coffee break |
| 11:45-12:45 | *Alain Pajor*<br>The RIP property for som*e models of random matrices II* |
| 12:45-14:00 | Lunch  at ESIEE |
| 14:00-15:30 | Djalil Chafai<br>*Behavior of the largest and smallest singular values of random matrices III* |
| 15:30-16:00 | Tea break |
| 16:00-17:00 | Alain Pajor<br>*The RIP property for some models of random matrices III* |

| Thursday | ENPC building (amphi Navier) |
|---|---|
| 09:30-10:00 | Coffee |
| 10:00-11:00 | Olivier Guédon<br>*Empirical methods and selection of characters II* |
| 11:00-11:15 | Coffee break |
| 11:15-12:45 | Guillaume Lecué<br>*Basic tools from empirical processes theory applied to the compress sensing problem III* |
| 12:45-14:00 | **Buffet at the Math Department (Copernic building fourth floor)** |

| Friday | ESIEE building (amphi 110 main floor) |
|---|---|
| 09:30-09:45 | Coffee |
| 09:45-11:15 | Olivier Guédon<br>*Empirical methods and selection of characters III* |
| 11:15-11:30 | Coffee break |
| 11:30-13:00 | Shahar Mendelson<br>*Applications of chaining methods III* |
| 13:00-14:00 | Lunch  at ESIEE |

# Campus de la Cité Descartes

Université Marne-la-Vallée

Direction Marne-la-Vallée Chessy

Ligne R.E.R. A

Direction Metz-Nancy

Direction Paris

Sortie Centre commercial-Champy

Sortie vers Cité Descartes

Arrêt Archimède n° 212,213,312

Bus

Bd Archimède

Bd Newton

Bd Copernic

Hôtel IBIS

Restau. U

Avenue Ampère

Bus n°213

La Poste Services centraux

E.S.I.E.E.

Gymnase n°213

Bus n°213

Avenue du Champy-Nesles

Gare de Noisy-Champs

Allée Jean Renoir

Centre Albert Camus

Allée Vaugirard

Allée du Promontoire

rue de la Butte

I.F.I.

Ecole Louis Lumière

Rue des Hauts Châteaux

rue Verte

Ecole d'Architecture

Rue Pascal

Blaise

Lumière

Avenue

Bd Descartes

Bd Copernic

ENSG

E.N.P.C.

Copernic

Restau. U

Rue des Frères

Rue Nobel

I.F.U.

Ingénieurs 2000

Bd. Albert Einstein

Rue Galilée

Ferme de la Haute Maison

Bd Descartes

Clément Ader

Rabelais

Lavoisier

Bois de l'étang

Arrêt Université n°213 Bus

Bus

I.U.T.

Bd Blaise Pascal

RN 370

Autoroute A4

Entrée A4

Sortie Champs-sur-Marne

Sortie Noisy-le-Grand

Direction Paris

N E O S

Distance 0 m 100 m 200 m

## BATIMENTS DE L'UNIVERSITE

1- **Copernic** : 5 Bd Descartes Champs-sur-Marne 77454 Marne-la-Vallée cedex2.

2- **I.U.T. site de Champs-sur-Marne** : Cité Descartes 77420 Champs-sur-Marne.

3- **Bois de l'Etang** : rue Galilée - Cité Descartes - 77420 Champs-sur-Marne.

4- **Lavoisier** : rue Galilée - Cité Descartes - 77420 Champs-sur-Marne

5- **François Rabelais**

6- **Clément Ader**

7- **Institut Francilien d'Ingénierie (I.F.I.)** : 2 Allée du Promontoire 93160 Noisy-le-Grand

8- **Centre Albert Camus**