Estimation of the Effective Dimension Reduction Subspace

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The aim of this talk is to introduce a new procedure providing an estimator of the effective dimension reduction (EDR) subspace in the multi-index regression model with deterministic design and additive noise. More specifically, the problem of estimating the projection matrix $\Pi^* = \Theta\,\Theta^\top$ based on the observations $(x_1, Y_1), \ldots, (x_n, Y_n)$ coming from the model

$$Y_i = f(x_i) + \varepsilon_i = g(\Theta^\top x_i) + \varepsilon_i, \qquad i = 1, \ldots, n,$$

is addressed. In the general setup we are interested in, the covariates $x_i \in \mathbb{R}^d$, Θ is a $d \times m^*$ orthogonal matrix ($\Theta^\top\Theta = I_{m^*}$) and $g : \mathbb{R}^{m^*} \to \mathbb{R}$ is an unknown function. To be able to estimate Π^* consistently, we assume that $\mathcal{S}^* = Im(\Theta)$ is the smallest subspace satisfying $f(x_i) = f(\Pi_{\mathcal{S}} x_i)$, $\forall i = 1, \ldots, n$, where $\Pi_{\mathcal{S}}$ stands for the orthogonal projector in $\mathbb{R}^d$ onto the subspace $\mathcal{S}$. We will focus our attention on the case where m^* is known.

Many methods dealing with the estimation of the EDR subspace perform principal component analysis on a family of vectors, say $\hat\beta_1, \ldots, \hat\beta_L$, nearly lying in the EDR subspace. This is in particular the case for the structure-adaptive approach proposed by Hristache, Juditsky, Polzehl and Spokoiny (*Ann. Statist.* 2001). In contrast with this approach, we propose to estimate the projector onto the EDR subspace by the solution to the optimization problem

$$\text{minimize} \quad \max_{\ell=1,\ldots,L} \hat\beta_\ell^\top (I - A)\hat\beta_\ell \quad \text{subject to} \quad A \in \mathcal{A}_{m^*},$$

where $\mathcal{A}_{m^*}$ is the set of all symmetric matrices with eigenvalues in $[0, 1]$ and trace less than or equal to m^*, with m^* being the true structural dimension. Under mild assumptions, $\sqrt{n}$-consistency of the proposed procedure is proved (up to a logarithmic factor) in the case when the structural dimension is not larger than 4. Moreover, the stochastic error of the estimator of the projector onto the EDR subspace is shown to depend on L logarithmically. This enables us to use a large number of vectors $\hat\beta_\ell$ for estimating the EDR subspace. The empirical behavior of the algorithm is studied through numerical simulations.