Four Different Problems: Interplay Between Geometry and Physics

1. **Sphere Packing Problem**

   - **Applications**: **low-temperature states of matter (liquids, crystals and glasses), granular media, biological media, communications, string theory, etc.**

2. **Number Vari3.802 130.890.145533(r)0.eroblem**

Interaction Energies of Many-Particle Systems

- **Total potential energy** $\Phi(\mathbf{r})$ **of** N **identical particles with positions**

  $\mathbf{r} \equiv \mathbf{r}$

Reformulations of the Covering and Quantizer Problems

Sphere Packing Problem

- **The packing density φ is the fraction of space** R 191]TJ 52007Á

Minkowski Lower Bound on φ

Random Packings Beat Checkerboard Lattice in Relatively Low

Ghost RSA Packin0.0398074402(g0.181173(c):)-309.828

Ghost RSA Packing: An Exactly Solvable Model

- At small times or, equivalently, low densities, can show

$$g_2(r;\varphi) = \Theta(r-1) + O(\varphi^3).$$

- At the maximum density $\varphi(\infty) = 1/2$,

$$g_2(r;\infty) \equiv \lim_{t\to\infty} g_2(r;t) = \quad \Theta(r -$$

Existence of Disordered Packings in High Dimensions

Disordered Packings Might Win in High Dimensions

Disordered Packings Might Win in High Dimensions

Hyperuniformity and Number Theory

- Surface-area coefficient enables rank ordering of hyperuniform point patterns.

Table 2:

Covering Problem

- The covering density associated with A_d^* at unit number density

Table 3: Best known solutions to the covering problem in selected dimensions.

Dimension, d	Covering	Covering Density, θ
1	$\mathbf{A}_1^{\square} \equiv \mathbb{Z}$	1
2		

Quantizer Problem

Nearest-Neighbor Functions

Series Representations

- **For example, for an ensemble,**

$$\mathbf{E_V}(\mathbf{R}) = \text{[illegible]} + \sum_{k=1}^{\infty} (-\text{[illegible]})^{k} \frac{\rho^{k}}{k}$$

Covering and Quantizer Calculations Using $\mathsf{E}_{\mathsf{V}}(\mathsf{R})$

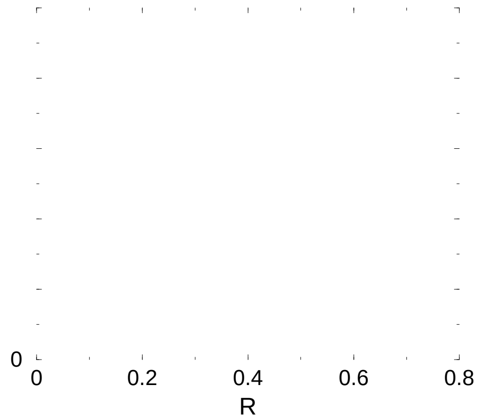

Table 6:

Bounds on the Quantizer Error

- **Revisiting Zador's Bounds (1982):**

$$\frac{\quad}{(d+2)\pi}$$ [redacted] $d/2)^{2/d} \le G_{min} \le$

Table 7:

RSA Quantizers

CONCLUSIONS

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