Large permutation invariant matrices are asymptotically free over the diagonal

Camille Male

Institut de Mathématiques de Bordeaux & CNRS

16 mai 2018

Free probability probability :

1. Generalizes classical probability : Free independence and associated CLT, cumulants, entropy, harmonic analysis...
2. Robust for the spectral analysis of large random multi-matrix models : e.g. unitarily invariant random matrices and Wigner matrices.

Traffic probability : to accommode models beyond this scope.

1. Generalizes non-commutative probability : a single independence which unifies the three non-commutative notions.
2. Permutation invariance is the canonical model of traffic independence in the large N limit.

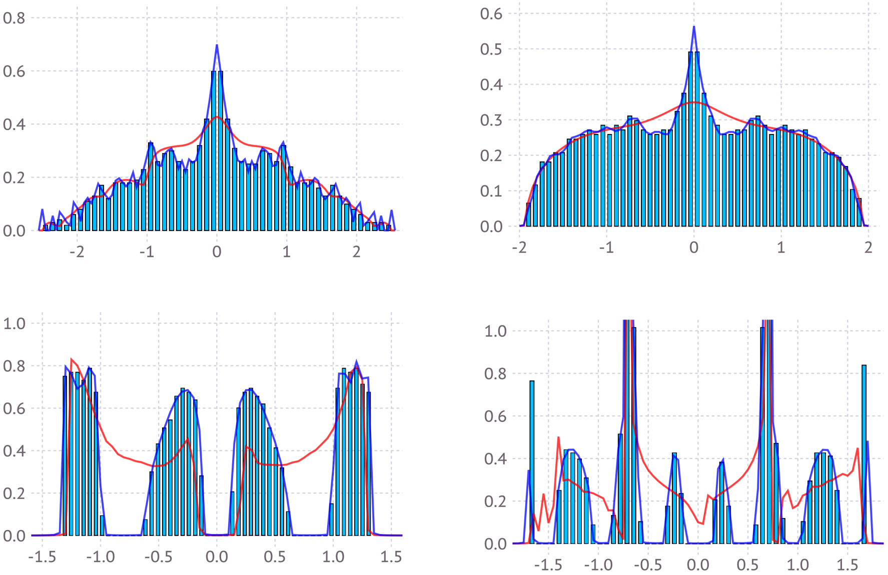
We show in the context of large random matrices that Voiculescu's notion of **conditional expectation provides an analytic tool for traffic independence**.

We write $\mathcal{M}_N$ for the set of N by N matrices, $\mathcal{D}_N \subset \mathcal{M}_N$ for the subset of diagonal matrices, $\Delta : \mathcal{M}_N \to \mathcal{D}_N$ for the diagonal map.

Theorem (Au, Cébron, Dahlqvist, Gabriel, M.)

Let $\mathbf{A}_{N,1} = (A_{N,1}^{(k)})_{k \in K}, \ldots, \mathbf{A}_{N,L} = (A_{N,L}^{(k)})_{k \in K}$ be independent families of random matrices which are uniformly bounded in operator norm and permutation invariant. Then $\mathbf{A}_{N,1}, \ldots, \mathbf{A}_{N,L}$ are asymptotically free over the diagonal in the operator valued non-commutative probability space $(\mathcal{M}_N, \mathcal{D}_N, \Delta)$.

Diagonal version of the usual fixed point equations remains valid $\Rightarrow$ numerical method


0.8
0.6
0.4
0.2
0.0
-2
-1
0
1
2
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-2
-1
0
1
2
1.0
0.8
0.6
0.4
0.2
0.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
1.0
0.8
0.6
0.4
0.2
0.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5