Stationary KPZ Fluctuations For the Stochastic Higher Spin Six Vertex Model

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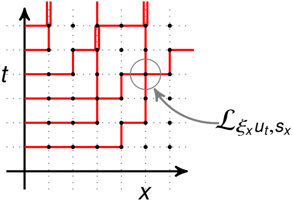
Stochastic Higher Spin Six Vertex Model [Corwin-Petrov '15]

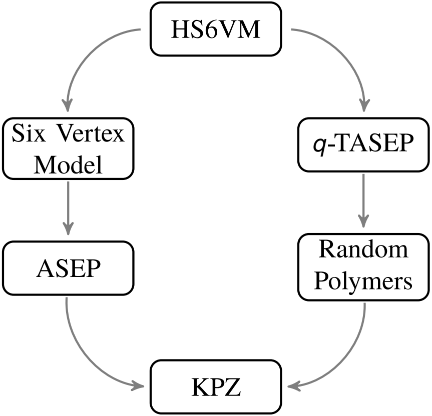
Boltzmann vertex weights

	g / g	$g-1$ / g	g / g	$g+1$ / g
$\mathcal{L}_{\xi u,s}$	$\frac{1-s\xi u q^g}{1-s\xi u}$	$\frac{-s\xi u+s\xi u q^g}{1-s\xi u}$	$\frac{-s\xi u+s^2 q^g}{1-s\xi u}$	$\frac{1-s^2 q^g}{1-s\xi u}$

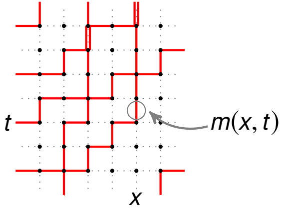
Construct a measure on the set of directed path on $\mathbb{Z}^2_{\geq 1}$



Map of principal degenerations of the HS6VM



In its stationary state the HS6VM can be defined on the full lattice $\mathbb{Z}^2$



Stationary product measure

$$\mathbb{P}(m(x,t) = M) \propto \left(\frac{\rho}{s_x \xi_x}\right)^M \frac{(s_x^2;q)_M}{(q;q)_M}$$

An important observable is the stationary height $\mathcal{H}$

$\mathcal{H}(x,t) - \mathcal{H}(x+\Delta x, t) =$ # of paths in $[x, x+\Delta x]$ at time t,

$\mathcal{H}(x, t+\Delta t) - \mathcal{H}(x,t) =$ # of paths crossing x

during the time interval $[t, t+\Delta t]$.

Exact formulas for the statistics of $\mathcal{H}$ are a consequence of

- Yang Baxter equation

$$L_{\frac{u_1}{u_2\sqrt{q}},\frac{1}{\sqrt{q}}} * L_{u_1,s} * L_{u_2,s} = L_{u_1,s} * L_{u_2,s} * L_{\frac{u_1}{u_2\sqrt{q}},\frac{1}{\sqrt{q}}}$$

- Elliptic determinants

$$\frac{\overline{\Theta}(\zeta A/Z)}{\overline{\Theta}(\zeta)} \frac{\prod_{1\le i<j\le n}\overline{\Theta}(a_i/a_j)\overline{\Theta}(z_j/z_i)}{\prod_{i,j=1}^{n}\overline{\Theta}(a_i/z_j)} = \det_{i,j=1}^{n}\left(\frac{\overline{\Theta}(\zeta a_i/z_j)}{\overline{\Theta}(\zeta)\overline{\Theta}(a_i/z_j)}\right)$$

We obtain

$$\left\langle \frac{1}{(\zeta q^{\mathcal{H}(x,t)};q)_\infty} \right\rangle$$

$$= \frac{1}{(q;q)_\infty} \sum_{k\geq 0} \frac{(-1)^k q^{\binom{k}{2}}}{(q;q)_k} \det\left(1 - f_{\zeta q^{-k}} A\right) G(\zeta q^{-k}),$$

with

$$f(n) = \frac{1}{1-q^n/\zeta},$$

$$A(n,m) = \frac{1}{(2\pi i)^2} \int_D \frac{dw}{w} \int_C dz \frac{z^m}{w^n} \frac{\exp\{xg(z)\}}{\exp\{xg(w)\}} \frac{(q\frac{v}{w};q)_\infty}{(q\frac{v}{z};q)_\infty} \frac{1}{z-w},$$

and G has a more complicated expression.

Our formulas are good for asymptotic analysis!

Theorem (IMS)

$$\frac{\mathcal{H}(x,\kappa x)-\eta x}{\gamma x^{1/3}} \xrightarrow[x\to\infty]{\mathcal{D}} F_{BR}.$$

Here F_{BR} is the Baik-Rains distribution [Baik-Rains'00].