Topological Recursion

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Let $\rho(\lambda)$ be the eigenvalue density or empirical spectral measure.

$$W_1(x) := \sum_{k=0}^{\infty} \frac{m_k}{x^{k+1}}, \quad m_k := \int_{\mathbb{R}} \lambda^k \rho(\lambda)\,\mathrm{d}\lambda$$

is the resolvent. Along with analogues $W_n(x_1, \ldots, x_n)$, satisfies recursion:

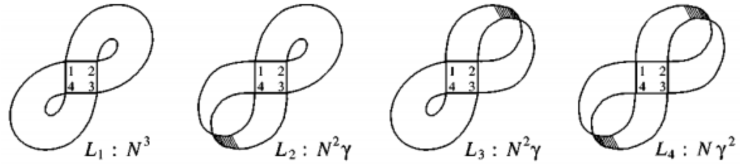
$$\kappa W_{n+2}(x, x, I) + \kappa \sum_{J \subseteq I} W_{|J|+1}(x, J) W_{|I-J|+1}(x, I - J)$$
$$+ \sum_{i=1}^{n} \frac{\partial}{\partial x_i} \frac{W_n(x, I - \{x_i\}) - W_n(I)}{x - x_i} + (\kappa - 1) \frac{\partial}{\partial x} W_{n+1}(x, I)$$
$$= \kappa N \left(V'(x) W_{n+1}(x, I) - P_n(x; I) \right), \quad I = (x_1, \ldots, x_n).$$

And Beyond

- Inverse Stieltjes Transform → Smoothed density

$$\tilde{\rho}(\lambda) = \frac{1}{2\pi \mathrm{i}} \lim_{\epsilon \to 0} \left[W_1(\lambda - \mathrm{i}\epsilon) - W_1(\lambda + \mathrm{i}\epsilon) \right]$$

- What makes it *topological*?



- Accessible derivation for Gaussian, Laguerre, Jacobi (*Aomoto's method: integration by parts, etc.*)
- Spectral curves and enumerative geometry (*The Eynard-Orantin generalisation, Seiberg-Witten representation, pairs-of-pants decomposition, etc.*)