Random-matrix behavior in the energy spectrum of the Sachdev-Ye-Kitaev model and in the Lyapunov spectra of classical chaos systems

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SYK model

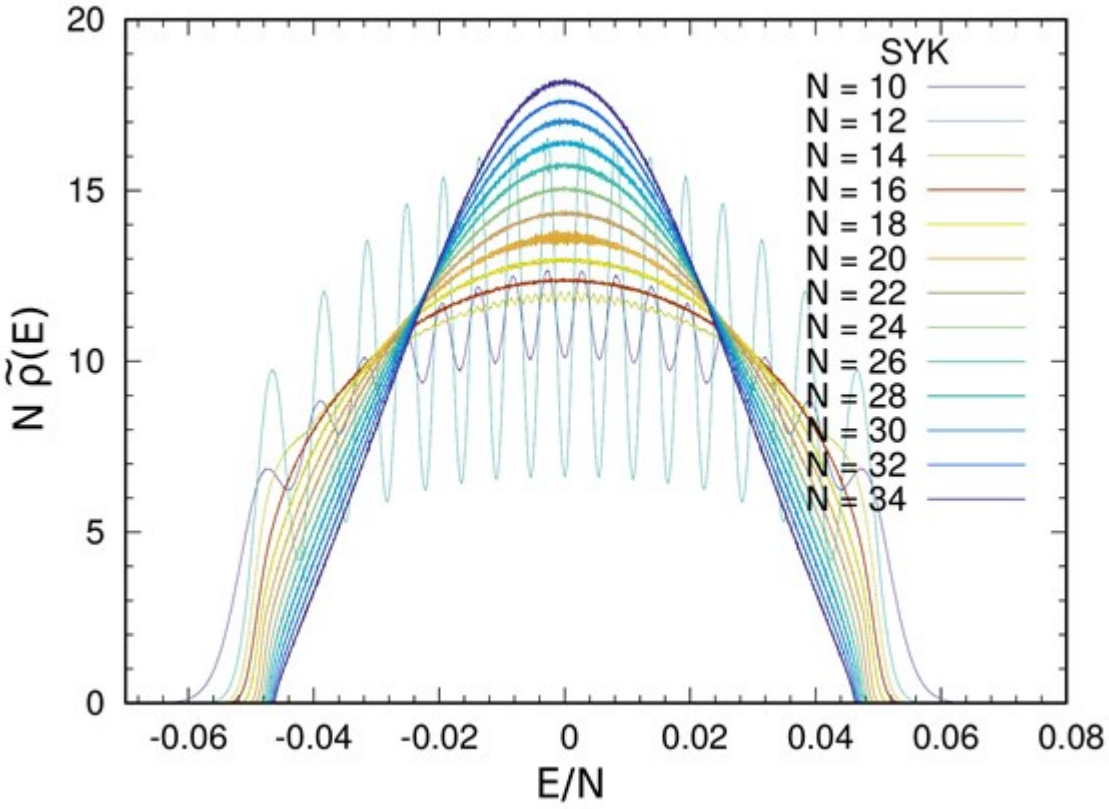
J. S. Cotler, …, MT, JHEP **1705**, 118 (2017)
(arXiv:1611.04650)

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a < b < c < d \le N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

[A. Kitaev: talks at KITP (Apr 7 and May 27, 2015)]

1. Solvable at large-N (strong coupling when $\beta J >> 1$), finite entropy / N at $T \to 0$
2. Holographically corresponds to 1+1D black holes
3. Satisfies the chaos bound

"Fast quantum information scrambler"
(Conjectured upper bound of the Lyapunov exponent $\lambda_{\mathrm{L}} = 2\pi k_{\mathrm{B}} T/\hbar$ realized, as in black holes)



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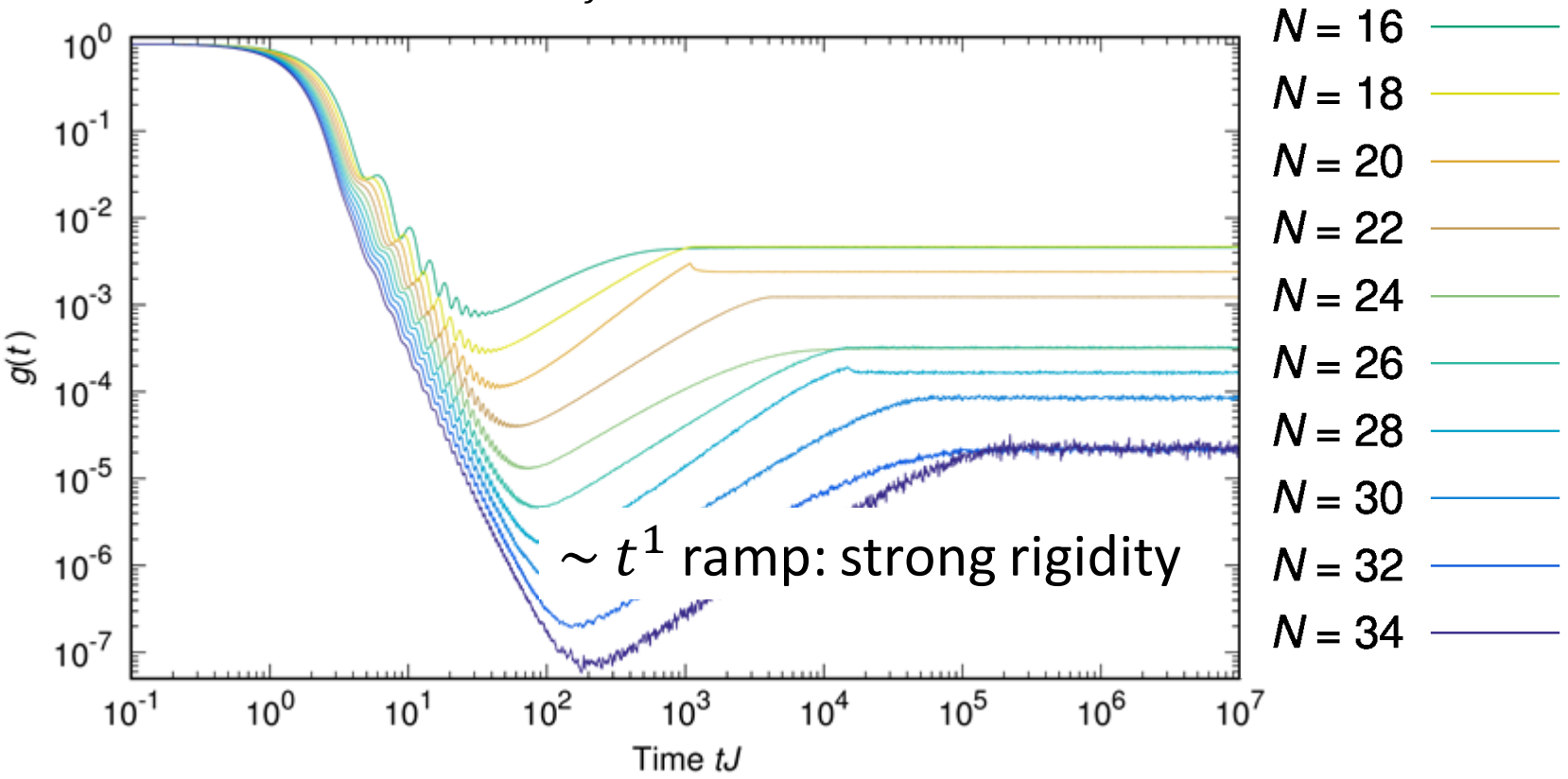
[You, Ludwig, Xu 2017]

$N_\chi \pmod 8$	0	1	2	3	4	5	6	7
qdim	1	$\sqrt{2}$	2	$2\sqrt{2}$	2	$2\sqrt{2}$	2	$\sqrt{2}$
lev. stat.	GOE	GOE	GUE	GSE	GSE	GSE	GUE	GOE

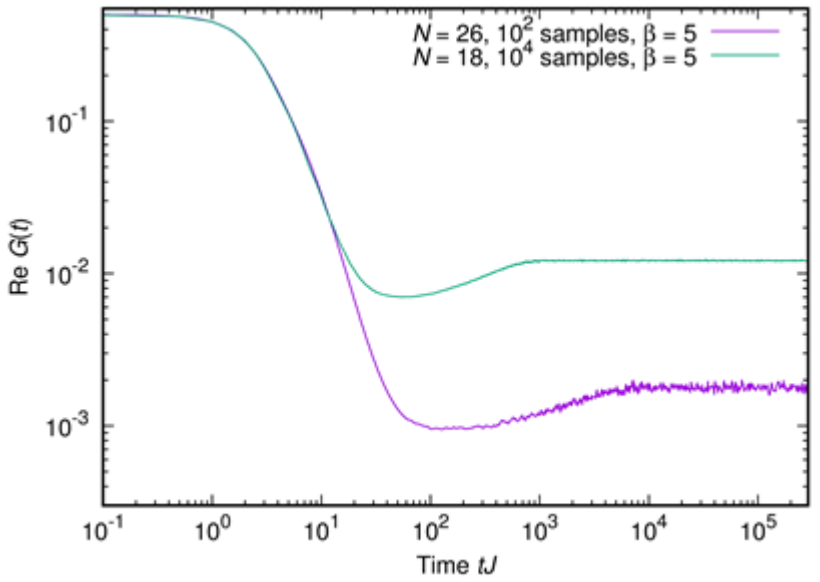
Spectral form factor

$$g(\beta,t)=\frac{\langle |Z(\beta,t)|^2\rangle_J}{\langle Z(\beta)\rangle_J{}^2} \qquad Z(\beta,t)=\mathrm{Tr}\left(\mathrm{e}^{-\beta\hat{H}-\mathrm{i}\hat{H}t}\right)$$



$$G(t)=\langle \chi_a(t)\chi_a(0)\rangle$$

Dip-ramp-plateau structure similar to $g(\beta,t)$ for $N\equiv 2 \pmod 8$



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in the energy spectrum of the Sachdev-Ye-Kitaev model
and in the Lyapunov spectra of classical chaos systems

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M. Hanada, H. Shimada, and MT,
Phys. Rev. E **97**, 022224 (2018) (arXiv:1702.06935)

Deviation at t initial infinitesimal deviation

$$\delta\phi_i(t) = T_{ij}\delta\phi_j(0)$$

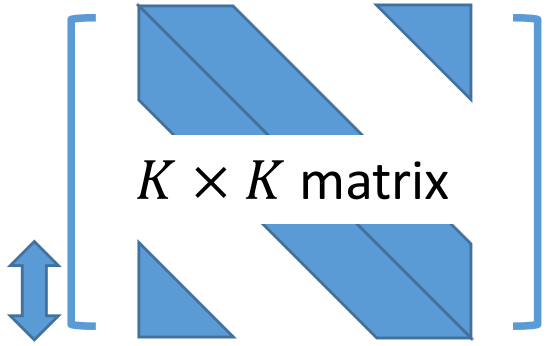
Singular values of T_{ij}: $\{a_k(t)\}_{k=1}^{K}$

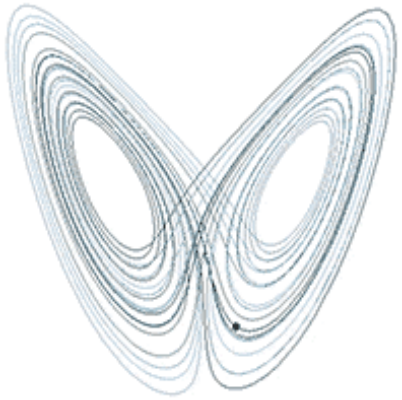
Time-dependent Lyapunov spectrum

$$\left\{\lambda_k(t) = \frac{\log a_k(t)}{t}\right\}_{k=1,2,\ldots,K}$$

Spectral correlation in $\lambda_k(t)$ observed for various classical chaos systems:
Logistic map, Lorenz attractor, etc.

Random matrix product





Ongoing work

Quantum chaos systems e.g. the SYK model:
Definition of Lyapunov spectra and study of its behavior